Color Computation: Consistency or Accuracy?

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Abstract

The CIE has compiled color matching functions (CMFs) at one nm sampling to be used for computational color. Since most instruments produce sampled data at resolutions considerably coarser than one nm, the recorded data must be interpolated to finer resolution before the multiply and sum required to compute the tristimulus values. This paper points out that the interpolation which should be used depends on the characteristics of the CIE interpolations which produced the CMFs, the characteristics of the reflectance signals and the noise associated with the recording instrument.

1 Introduction

The reason for computational color is to facilitate the communication of color description. If people in different places can describe color numerically in a consistent manner, then there is a basis for defining and satisfying color specifications. The CIE has made recommendations on how to numerically compute tristimulus values, [1]. Consistent results are obtained when these recommendations are followed. There is a question of accuracy. Does the value computed agree with some physical or psychophysical characteristic of an object under investigation?

To illustrate the difference between consistency and accuracy, consider the following examples. A printer must reproduce a standard digital image within a given tolerance specified in terms of CIE Delta-Eab. The manufacturer prints the target before shipment, measures it, computes the CIELAB values and finds the target within tolerance. The user tests the printer after receiving it using the same instruments and procedure. If the result is in tolerance the device is accepted and the invoice is paid. If the result is out of tolerance, the user must convince the manufacturer that his results are correct. If the user has duplicated the instruments and procedure of the manufacturer, there is no question. Of course, the assumption is that everything is in calibration.

Suppose a researcher attempts to determine the tolerance of a viewer to slight color differences. Incidentally, this example could substitute a scanner for a viewer. The viewer's CMFs are measured at some sampling interval. The reflectances of samples are measured at some, possibly different sampling interval. The sampled data is interpolated to a common, finer sampling interval, multipled and summed to estimate the tristimulus values. The question here is accuracy. Does the computed result represent the process of the viewer? Note that the answer cannot be answered with certainty since it is impossible to know how well the interpolations actually represent the continuous signals. It is possible to optimize the sampling and interpolation process given assumptions about the signals. The guidelines for this optimization are given in this paper.

As a practical matter, the results presented here show the limitations for accuracy of the tabulated CIE color matching functions, which are the foundation of computational color. The results also show that following the exact CIE recommendations is the best way to assure consistent if not accurate results.

2 Ideal Sampling and Interpolation

To design or test color devices computer simulations are performed. Usually the simulation includes computation of CIE tristimulus values. In any case, the accuracy of the simulations in modeling analog devices depends upon correct sampling and processing of the sampled data. The data for the simulations usually comes from instruments which measure the spectral reflectance or radiance of a colored object. This sampled signal is then used to obtain the ideal color values by digital computation.

Unfortunately, color signals are not perfectly bandlimited. This means that any sampling will result in some error. The statistics of the error will depend upon the relationship between the sampling and reconstruction (interpolation) methods and the statistical characterization of the color signals.

The exact method used to construct the CIE color matching functions is not published. However, it is sufficient to demonstrate the effect of sampling and interpolation for a general case. In the next section, examples of real signals will be used to indicate practical uses. The general sampling operation can be described by

$$r_s(n) = \int_{-\infty}^{\infty} r_a(t) s_n(t) dt \tag{1}$$

where $r_a(t)$ represents the analog signal in the continuous domain (reflectance or radiance) and $s_n(t)$ represents an arbitrary sampling function or aperture. Classical sampling is described by $s_n(x) = \delta(x - n)$.

Let $\mathbf{r}$ be a signal defined in an N-dimensional space, Ω_N, where vectors are defined as column vectors. The above sampling can be considered a subsampling of the space. The integral in the continuous case can be considered an inner product operator. The general sampling can be represented by

$$r_s(n) = \mathbf{r}^T \mathbf{s}_n = < \mathbf{r}, \mathbf{s}_n > \tag{2}$$

where $\mathbf{s}_n$ is the n^{th} sampling vector. The resulting samples are combined into a vector which corresponds to a sampled signal

$$\mathbf{r}_s = [r_s(1), r_s(2), \cdots, r_s(M)]^T \tag{3}$$

The sampling reduces the dimensionality of the space from N to M. The sampled signal can be represented by a linear operation

$$\mathbf{r}_s = \mathbf{S}^T \mathbf{r} \tag{4}$$

where $\mathbf{S}$ is an $N \times M$ matrix and $\mathbf{r}_s$ is a $M \times 1$ vector. The vector $\mathbf{r}_s$ which contains the sample values represents a vector in the M dimensional subspace of Ω_N spanned by the columns of $\mathbf{S}$. A reconstruction (interpolation) method is a mapping from a lower dimensional space defined by samples to a higher dimensional space. For example, using eq.(4)

$$\mathbf{r}_i = \mathbf{H}_M \mathbf{r}_s = \mathbf{H}_M \mathbf{S}^T \mathbf{r} \tag{5}$$

the interpolation matrix, $\mathbf{H}_M$ is $N \times M$. The interpolation lies in a subspace defined by the columns of $\mathbf{H}_M$, $\mathbf{r}_i \in \Omega_H$. From this, it is easy to see that the interpolation matrix (method) should be chosen to best represent the signal $\mathbf{r}$. In other words, the signal should have most of its power in Ω_H. Problems can arise when different interpolation methods are used in the computation process. The use of different methods is subtle in cases of tabulated values such as those given by the CIE. It is unclear what interpolation method was used to generate the tables. It can be shown that optimum interpolation requires matching the characterisitics of the signal and interpolation [3].

It is common to choose interpolation operators which can exactly interpolate themselves, which means that $\mathbf{HSH} = \mathbf{H}^T$. This, in turn, implies that $\mathbf{H}$ is a projection operator. However, the projection need not be orthogonal. The reconstruction is exact if $\mathbf{r} \in \Omega_H = \Omega_S$. In this case, $\mathbf{P} = \mathbf{HS}(\mathbf{S}^T\mathbf{S})^{-1}\mathbf{S}^T$ is the identity operator on Ω_H.

Errors can arise is several ways. One is that the sampling and interpolation are not matched, i.e. $\Omega_H \neq \Omega_S$. A second is that $\mathbf{r}$ is not completely contained within Ω_H. A combination of the two is the most likely case. The fraction of the signal power of $\mathbf{r}$ in Ω_H^c is a measure of the error where $\Omega_N = \Omega_H^c \times \Omega_H$.

The results of using several interpolation methods is presented in Table 1. For this experiment, simulated color matching functions and radiant spectra were generated by controlling the amount of power in and out of the spaces defined by the interpolation methods. The model for the reflectance spectra is given by

$$\mathbf{r} = \mathbf{P}_H \mathbf{f} + \sigma \mathbf{P}_H^c \mathbf{f} \tag{6}$$

where $\mathbf{f}$ is a random vector, $\mathbf{P}_H$ is the projection onto the subspace defined by the interpolation matrix, $\mathbf{P}_H^c$ is the projection onto the complement of Ω_H, and σ is a weighting factor determining the relative amount of power outside the subspace. The parameter σ determines how well the signal characteristics are matched to the interpolation. The simulated CMFs were generated using a similar model.

For the experiment, three common interpolations were used: the bandlimited interpolation given by a windowed sinc function, the Laplacian interpolation recommended by the CIE and the linear interpolation. The sampling and interpolation was done at a 4:1 ratio of the original full resolution signal. For the table presented here, the CMFs were generated using the $\mathbf{H}$ defined by the bandlimited space. Simulated tabulated CMF vectors were generated by sampling the simulated CMF and interpolating them using the Laplacian method. The radiant spectra were sampled and reconstructed using all three interpolation methods.

For the evaluation, the following values were computed. The actual tristimulus value was computed; call this $t = \mathbf{a}^T\mathbf{r}$. Note for this example, it might be referred to as a monostimulus value. The tristimulus value obtained using the full resolution radiant spectra and the tabulated (Laplace interpolated) CMF. This

value attempts to simulate what can be done by researchers using high resolution spectraradiometers to obtain data. The researcher is limited by the CIE tabulated values for the CMFs but can generate original full resolution radiant spectra. This would be used to establish the estimated actual value; call this $t_s = \mathbf{a}_l^T \mathbf{r}$. The tristimulus values estimated by the various interpolations are given by $t_b = \mathbf{a}_l^T \mathbf{r}_b$, $t_l = \mathbf{a}_l^T \mathbf{r}_l$, $t_c = \mathbf{a}_l^T \mathbf{r}_c$ for the bandlimited, Laplacian and linear interpolations respectively. In addition, the tristimulus values obtained by using the actual CMF and the various interpolations were computed, $t_{ab} = \mathbf{a}^T \mathbf{r}_b$, $t_{al} = \mathbf{a}^T \mathbf{r}_l$, $t_{ac} = \mathbf{a}^T \mathbf{r}_c$.

Table 1 shows the following comparisons under various amounts of out-of-band power for both the CMFs and radiant spectra. The mean square error (MSE) of the interpolation of the radiant spectra was computed to show how well the interpolation works. The MSE of the tristimulus estimates and t_s indicates the error obtained in simulating a system using the tabulated values. Finally, the MSE of the tristimulus estimates using the actual CMF and t indicates the actual error.

From the table, it is seen that it is possible to have more accurate estimates of the radiant spectra but worse estimates of the simulated tristimulus values, e.g. the case of $\sigma_{cmf} = 0.05$ and $\sigma_r = 0.02$. In general, for cases where there is little out-of-band power, the bandlimited interpolation is optimal. Where the out-of-band power is higher, the optimal interpolation is the same as that used to obtain the tabulated CMF. This was tested by varying the interpolation method for the CMF and rerunning the experiment. This effect explains the results of the simulation comparison of bandlimited and Laplacian interpolators presented in [2].

3 Results on Radiant Spectra

There is a question of which out-of-band power case best represents that encountered by workers in the field. While there is no definitive answer, an experiment was done to indicate the possibilities. It is impossible to determine the actual CMFs which would be required to test how much power is out-of-band. It is possible to collect high resolution radiant spectra which can be assumed to be full resolution. This data can be subsampled and then interpolated to test which method works the best. Spectral data was used from a well known database. This data was sampled at 2 nm. Only three data sets were available. In order to obtain results for a wider variety of data, additional data sets were generated by subsampling the three original sets. Subsampling the original spectral data at 4:1 results in a simulated sampling of 8 nm resolution. This was included to demonstrate that optimal interpolations depend on the character of the data. The results of interpolating various subsampled spectra at 4:1 from various ensembles is presented in Table 2. The first column of the table shows the simulated resolution before and after interpolation. The mean square error was taken over the midrange of the signals to avoid end effects.

The results show that no one interpolation method works best in all cases. The linear interpolation does surprisingly well. The bandlimited interpolation is never the best of the three tested and never outperforms the linear interpolation. The Laplacian outperforms the linear in two cases and then only by a slight amount. Comparing this to previous work, it should be noted that the results of the comparison of integration methods used to compute tristimulus values, presented in [4], may be biased by the use of the tabulated (interpolated) CIE CMF values.

4 Summary

It has been shown that the characteristics of the signals determine the optimal interpolation method for color computations from sampled data. The experiments with actual radiant spectra indicate that for high resolution data, linear interpolation is often the preferred method. This should be taken into account during the next revision of the CIE recommentations and tabulation of color matching functions. It has also been shown that for many realistic representations of signals, the recommended Laplace interpolation is optimal if it is assumed that the tabulated CMFs were obtained using that method or one very similar.

References

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[4] F. W. Billmeyer, Jr. and H. S. Fairman, "CIE Method for Calculating Tristimulus Values," Color: R. & A., Vol. 12, No. 1, pp. 27-36, Feb. 1987.

Table 1: Mean Square Errors in Spectra and Tristimulus Values for Interpolators

σ_{cmf}	σ_r	$r-r_d$	$r-r_l$	$r-r_c$	$t-t_d$	$t-t_l$	$t-t_c$	t_s-t_d	t_s-t_l	t_s-t_c
0.02	0.02	0.570	0.744	1.170	1.087	2.660	3.365	1.178	1.241	1.941
0.05	0.02	0.571	0.732	1.152	1.282	2.927	3.686	1.333	1.273	2.035
0.20	0.02	0.576	0.742	1.162	1.950	2.447	2.801	0.943	0.839	1.322
0.02	0.05	0.706	0.698	1.107	1.214	2.397	2.999	1.265	1.029	1.616
0.05	0.05	0.702	0.701	1.111	1.203	2.142	2.697	1.293	0.970	1.505
0.20	0.05	0.719	0.731	1.154	1.980	2.575	3.051	1.249	0.939	1.544
0.02	0.20	1.496	0.987	1.172	1.730	1.931	2.399	2.169	1.053	1.276
0.05	0.20	1.480	0.985	1.175	2.235	2.423	2.924	2.662	1.494	1.700
0.20	0.20	1.479	1.003	1.204	2.778	2.791	3.232	2.509	1.171	1.524

Table 2: Mean Square Errors in Actual Sampled Spectra for 4:1 Interpolation

resolution before:after (nm)	data set	$r-r_d$	$r-r_l$	$r-r_c$
8:2	objects	0.026	0.012	0.007
8:2	paint	0.035	0.012	0.007
8:2	munsell	0.035	0.015	0.016
16:4	objects	0.022	0.024	0.015
16:4	paint	0.026	0.024	0.013
16:4	munsell	0.028	0.028	0.019
32:8	objects	0.031	0.037	0.029
32:8	paint	0.056	0.062	0.053
32:8	munsell	0.051	0.047	0.049