Estimation of Illumination for Color Correction

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Abstract

The reproduction of color images often requires color correction. The orignal image was recorded under one illumination while the reproduction is to be viewed under another or made to appear as if it was recorded under another. Since it is often the case that the original illumination is unknown to the image scientist, it is necessary to estimate it. This paper presents methods of estimating the illuminant and discusses the sensitivity of the methods to various assumptions.

1. Introduction

It is often very difficult to control the lighting in a photographic situation. Images recorded under ambient illumination are biased by that illumination. The classic examples are the red shifts of photographs taken under incandescent lighting and green shifts of photographs taken under fluorescent lighting. It is often desirable to produce an image whose colors are indicative of the objects in the image or as the colors would appear under more uniform illumination. This paper presents two methods for estimating the spectral distribution of the illuminant and discusses their use in practical situations. The use of the illuminant to perform the actual correction is discussed elsewhere [1].

The color recording process can be described by the algebraic model

$$\mathbf{t} = \mathbf{M}^T\mathbf{L}\mathbf{f} \qquad (1)$$

where $\mathbf{f}$ is a vector of length N representing the reflectance (or transmittance) of an object at N wavelengths within the visible band, $\mathbf{L}$ is a diagonal matrix whose elements represent the power of the illuminant, $\mathbf{M}$ is an NxK matrix representing the sensitivity of the recording device or the transmission of scanning filters, and $\mathbf{t}$ represents the recorded values from which the color is to be reproduced. It is usual in photographic applications that $K = 3$. In satellite applications, the sensors are multispectral with many more bands. The elements of the reflectance vector and the illuminant matrix represent samples of a continuously varying function of wavelength. The vector $\mathbf{Lf}$ represents the rediant power reflected by the object at each sample wavelength. The elements of $\mathbf{t}$ represent the approximation to the integrated power of the radiation which passes through the scanning filters or is sensed by the detectors. Typical sampling is 10 nm in the visible spectrum from 400nm to 700nm which implies $N = 31$. It is noted that eq. (1) holds for each pixel in a 2-D image. It is assumed that the illuminant is constant over the region of interest.

Since the image is to be viewed by humans, the scanning filters, $\mathbf{M}$, should correspond (or be within a linear transformation of) the spectral sensitivities of the color sensors in the human eye (cones). The CIE, the international organization which defines color standards, has defined standard functions which satisfy this restriction. When using these standard color matching functions, the resulting elements of the vector $\mathbf{t}$ are called tristimulus values.

The restriction of $\mathbf{M}$ to be within a linear transformation of the sensitivity of the eye is predicated on the assumption that the objects which are viewed have a relatively unrestricted distribution of reflectances across the visible bandwidth. In cases where this assumption cannot be made, the scanning filters should be derived by considerations of the possible values of the objects. For example, when scanning photographic film, narrowband filters which correspond to the peak dye response are used. Some cases where the assumption is useful are found in television cameras and color copiers.

The scanning or viewing process greatly reduces the dimensionality of the vector space of the data. The problem of estimating the N diagonal elements of $\mathbf{L}$, from a collection of K dimensional vectors, $\{\mathbf{t}_i\}_{i=1}^{q}$ is greatly underdetermined. A solution is possible if the illumination can be restricted in some way. The fact that most real illuminant spectra are smoothly varying continous functions can be used to reduce the degrees of freedom in the estimation problem. In this paper, the illuminant will be assumed to be of a class that can be well represented by K principal components or basis vectors. This assumption is valid for many important lighting conditions including daylight and some incandescent lights [2]. Fluorescent lights which often contain sharp spectral peaks would not satisfy the assumptions. This assumption leads to the representation

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$$\mathbf{L} = \mathbf{L}_1 l_1 + \mathbf{L}_2 l_2 + \mathbf{L}_3 l_3 \tag{2}$$

where $\mathbf{L}_1$, $\mathbf{L}_2$, and $\mathbf{L}_3$, are NxN diagonal basis matrices. Now, to estimate $\mathbf{L}$ it is sufficient to estimate the three vector $\mathbf{l} = [l_1, l_2, l_3]$.

Previous work on illuminant estimation concentrated on the determination of the chromaticity coordinates of the light source. The chromaticity of a color with tristimulus values given by the vector $\mathbf{t}$ is the pair of values $\mathbf{c} = [t_1/(t_1 + t_2 + t_3), t_2/(t_1 + t_2 + t_3)]$. This "normalized" vector describes the "color" of the light or object, independent of its brightness. Papers by Lee, [3,4], describe methods using the reflection properties of objects to estimate the chromaticity of the source. Note that once the chromaticity is found, the illumination vector, $\mathbf{l}$, can be determined to within a multiplicative constant. Other work attempts to estimate both the reflectance and illuminant simultaneously [5].

2. Maximum Likelihood Estimation

The collection of color pixels in an image can be modelled by

$$\mathbf{t}_i = \mathbf{M}^T \mathbf{L} \mathbf{f}_i + \mathbf{n}_i \tag{3}$$

where we assume the pdf of $\mathbf{n}_i$ is zero mean Gaussian with covariance matrix $\mathbf{K_n}$. The noise is assumed to be independent of the reflection vectors.

In order to find the maximum likelihood (ML) estimate of the vector $\mathbf{l}$, the distribution of the tristimulus vectors conditioned on the illuminant must be known. The effects of various assumptions about the parameters of this pdf will be mentioned later. Again, the Gaussian form will be assumed for the vectors $\mathbf{f}_i$. The covariance matrix for the vectors are denoted $\mathbf{K_f}$. Since the vectors are nonnegative, they must have a nonzero mean denoted $\mathbf{m_f}$. It is noted that the reflection vectors, $\mathbf{f}_i$, are bounded between 0 and 1, which means that for a Gaussian probability to be reasonable the distribution should be heavily concentrated about the mean.

Using the standard approach of maximizing the logarithm of the likelihood function,

$$\max_{\mathbf{l}} \ln[p(\mathbf{t}_1, \mathbf{t}_2, ..., \mathbf{t}_s|\mathbf{l})] \tag{4}$$

Because of the multiplicative relation between the vectors $\mathbf{f}_i$ and $\mathbf{l}$ in equation (3), the solution to the maximization problem will yield a nonlinear system of equations given by

$$-\mathbf{K_l}^{-1}(\mathbf{l} - \mathbf{m_l}) - \frac{s}{2}\begin{pmatrix} Tr([\mathbf{A(l)K_f A}^T(\mathbf{l}) + \mathbf{K_n}]^{-1}\mathbf{M}^T\mathbf{L}_1\mathbf{K_f A}^T(\mathbf{l}) + \mathbf{A(l)K_f L}_1\mathbf{M}) \\ Tr([\mathbf{A(l)K_f A}^T(\mathbf{l}) + \mathbf{K_n}]^{-1}\mathbf{M}^T\mathbf{L}_2\mathbf{K_f A}^T(\mathbf{l}) + \mathbf{A(l)K_f L}_2\mathbf{M}) \\ Tr([\mathbf{A(l)K_f A}^T(\mathbf{l}) + \mathbf{K_n}]^{-1}\mathbf{M}^T\mathbf{L}_3\mathbf{K_f A}^T(\mathbf{l}) + \mathbf{A(l)K_f L}_3\mathbf{M}) \end{pmatrix} + 1/2\sum_{i=1}^{s}\begin{pmatrix} (\mathbf{c}_1 - \mathbf{A(l)m_f})^t[\mathbf{A(l)K_f A}^T(\mathbf{l}) + \mathbf{K_n}]^{-1}\mathbf{M}^T\mathbf{L}_1\mathbf{K_f A}^T(\mathbf{l}) \\ (\mathbf{c}_2 - \mathbf{A(l)m_f})^t[\mathbf{A(l)K_f A}^T(\mathbf{l}) + \mathbf{K_n}]^{-1}\mathbf{M}^T\mathbf{L}_2\mathbf{K_f A}^T(\mathbf{l}) \\ (\mathbf{c}_3 - \mathbf{A(l)m_f})^t[\mathbf{A(l)K_f A}^T(\mathbf{l}) + \mathbf{K_n}]^{-1}\mathbf{M}^T\mathbf{L}_3\mathbf{K_f A}^T(\mathbf{l}) \end{pmatrix}$$

$$\begin{pmatrix} +\mathbf{A(l)K_f L}_1\mathbf{M}[\mathbf{A(l)K_f A}^T(\mathbf{l}) + \mathbf{K_n}]^{-1}(\mathbf{c}_i - \mathbf{A(l)m_f}) - 2\mathbf{m_f}^T\mathbf{L}_1\mathbf{M}(\mathbf{A(l)K_f A}^T(\mathbf{l}) + \mathbf{K_n})^{-1}(\mathbf{c}_1 - \mathbf{A(l)m_f}) \\ +\mathbf{A(l)K_f L}_2\mathbf{M}[\mathbf{A(l)K_f A}^T(\mathbf{l}) + \mathbf{K_n}]^{-1}(\mathbf{c}_i - \mathbf{A(l)m_f}) - 2\mathbf{m_f}^T\mathbf{L}_2\mathbf{M}(\mathbf{A(l)K_f A}^T(\mathbf{l}) + \mathbf{K_n})^{-1}(\mathbf{c}_2 - \mathbf{A(l)m_f}) \\ +\mathbf{A(l)K_f L}_3\mathbf{M}[\mathbf{A(l)K_f A}^T(\mathbf{l}) + \mathbf{K_n}]^{-1}(\mathbf{c}_i - \mathbf{A(l)m_f}) - 2\mathbf{m_f}^T\mathbf{L}_3\mathbf{M}(\mathbf{A(l)K_f A}^T(\mathbf{l}) + \mathbf{K_n})^{-1}(\mathbf{c}_3 - \mathbf{A(l)m_f}) \end{pmatrix} = \nabla_{\mathbf{l}} p(\mathbf{l}|\mathbf{t}_1, \mathbf{t}_2, ..., \mathbf{t}_s) = 0$$

where $\mathbf{A(l)} = \mathbf{M}^T\mathbf{L}$. The solution to this system can be obtained by any of several methods found in [6]. The method used here was a modified Newton-Raphson scheme.

One problem with this method is that it may converge to any local extrema or saddle point. To insure convergence to a maximum, it is possible at each step to adjust the Hessian estimate so that it is negative definite. This is done by subtracting a constant diagonal matrix from the Hessian which will make the Hessian negative definite and well-conditioned. The iterative step becomes

$$\mathbf{l}_{k+1} = \mathbf{l}_k - [\hat{\nabla}^2_{\mathbf{l}} g(\mathbf{l}_k) - \alpha \mathbf{I}]^{-1} \nabla_{\mathbf{l}} g(\mathbf{l}_k) \tag{5}$$

where $g(\mathbf{l}) = \ln p(\mathbf{l}|\mathbf{t}_1, \mathbf{t}_2, ..., \mathbf{t}_s)$ and α is zero if $\hat{\nabla}^2_{\mathbf{l}} g(\mathbf{l}_k)$ is negative semidefinite. If $\hat{\nabla}^2_{\mathbf{l}} g(\mathbf{l}_k)$ is not negative semidefinite, then α is approximately equal to the largest positive eigenvalue of $\hat{\nabla}^2_{\mathbf{l}} g(\mathbf{l}_k)$. Further adjustments of α may be neccessary to keep equation (5) from becoming ill-conditioned.

3. Covariance Matching Method

Another method of estimating the illumination is to estimate the covariance matrix of the tristimulus values $\mathbf{K_t}$, and match this to a 3x3 matrix that is a function of $\mathbf{l}$.

Using the model of equation (3) the covariance can be written as

$$\mathbf{K_t} = \mathbf{M}^T\mathbf{L}\mathbf{K_f}\mathbf{L}\mathbf{M} + \mathbf{K_n} \tag{6}$$

An estimate of $\mathbf{K_t}$ from the tristimulus values can be made by

$$\hat{\mathbf{K}}_{\mathbf{t}} = \frac{1}{s}\sum_{i=0}^{s-1}(\mathbf{t}_i - \mathbf{m}_t)(\mathbf{t}_i - \mathbf{m}_t)^T \tag{7}$$

where $\mathbf{m}_t$ is the sample mean of the ensemble of the measured tristimulus vectors. It is assumed that $\mathbf{K_f}$ is a known NxN matrix, and $\mathbf{K_n}$ is a known 3x3 matrix. It is important to note that $\mathbf{m}_t$, the mean of $\mathbf{t}$, is assumed known. Knowledge of the mean of the tristimulus vectors is perhaps easier to gain than knowledge of the mean of the reflectance vectors. Substituting equation (2) into equation (6), we obtain

$$\mathbf{R_c} - \mathbf{R_n} = \mathbf{M^t L_1 R_f L_1 M} l_1^2 + \mathbf{M^t}[\mathbf{L_2 R_f L_1} + \mathbf{L_1 R_f L_2}]\mathbf{M} l_1 l_2 \quad (8)$$

$$+\mathbf{M^t L_3 R_f L_3 M} l_3^2 + \mathbf{M^t}[\mathbf{L_3 R_f L_1} + \mathbf{L_1 R_f L_3}]\mathbf{M} l_1 l_3$$

$$+\mathbf{M^t L_2 R_f L_2 M} l_2^2 + \mathbf{M^t}[\mathbf{L_2 R_f L_1} + \mathbf{L_1 R_f L_2}]\mathbf{M} l_2 l_3$$

Note that the matrices multiplying the scalar quantities $\{l_i\}$ are each 3x3 symmetric matrices so that six nonlinear equations are obtained which can be solved for the vector l. The system of equations is overdetermined with six equations and three unknowns. A solution was found by using a minimum mean square error approach. Again, a nonlinear system of equations must be solved. Methods which are similar to those used for the ML solution were used here.

4. Results

In order to evaluate the methods for illuminant estimation, several levels of tests were done. Since a spectraradiometer was not available, all tests were done by computer simulation. However, varying degrees of accuracy of the assumptions were tested. Recall that the important assumptions were knowledge of the mean of the distribution of reflectance vectors and the covariance structure of the distribution of the reflectances.

In the best of all cases where all the parameters are known, the only error is caused by the noise. The number of pixels will determine the accuracy of the estimate for a given SNR. For this case, both methods performed well. An example of the estimate for ten pixels and a 35dB SNR is shown in fig.1.

It is often assumed that the covariance matrix of the reflectances is diagonal, represented an uncorrelated process. In practice, this will rarely be the case. To test the effects of correlation, data sets were obtained from a sequence of color patches from a color copier and from the Kodak Wratten filter set. These data yielded highly correlated covariance structures. The actual covariance matrices do not closely approximate a stationary process in that the diagonal elements vary by a factor of 5. The means of the two ensembles were not uniform as one might hope if the integrated color of the ensemble were gray. The variation of the mean was about 50% of the average over the visible band.

Using the uniform estimate for the mean of the reflectances but with the correct covariance matrix gives the ML estimate shown in fig. 2. The covariance matching method produced similarly good results. Using the correct mean and a diagonal convariance matrix resulted in an extremely poor estimate. This indicates the importance of the covariance matrix. To determine the extent of the dependence on knowledge of the covariance, the matrix was approximated by an exponential decaying form. If this approximation was adjusted to account for the nonstationary character of the variation, the results were still quite good; the estimate from the covariance matching method is shown in fig. 3. For this case, the ML result was much worse which indicates some advantage of the covariance method. Using the exponential decaying covariance with a stationary assumption produced an unacceptably poor estimate for both methods.

The results of these and other tests indicate that the estimation of the illuminant is possible if the mean of the ensemble is reasonably well estimated and the variation by wavelength is known approximately. The use of the knowledge of the illuminant can then be used to perform color correction for various types of image reproduction needs.

5. References

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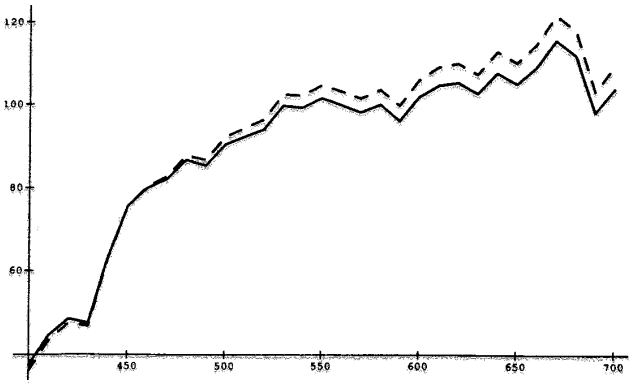

Figure 1

Comparison of actual (solid line) and estimated illuminants. Known mean and covariance structure.

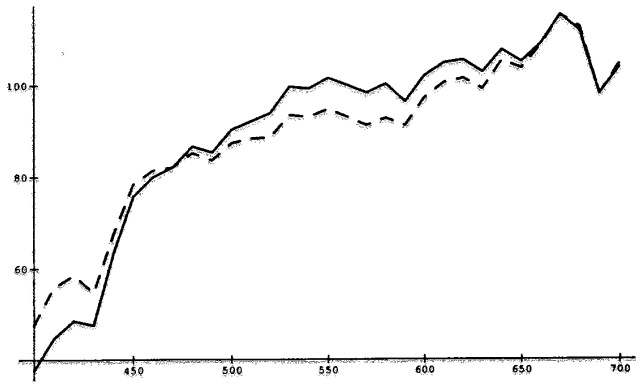

Figure 2

Comparison of actual (solid line) and estimated illuminants. Uniform mean and actual covariance structure.

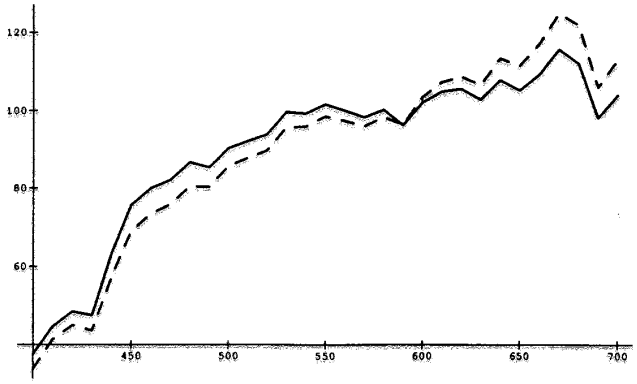

Figure 3

Comparison of actual (solid line) and estimated illuminants. Estimated mean and exponential covariance structure.